

# New Classess of Star Related Cubic Graceful Graphs

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**ABSTRACT:** A graph labeling is the assignment of labels, traditionally represented by integers, to the edges or vertices, or both of a graph. Formally, given a graph  $G=(V,E)$ , a vertex labeling is a function of  $V$  to a set of labels. In this paper, focus will be given on star related cubic graceful graphs. A graph which admits cubic graceful labeling is called cubic graceful graphs. We have proved that the graph  $pK_{1,q} \cup rK_{1,s}, \cup_{r=1}^t K_{1,r}, (K_{1,m}:G), (K_{1,p} \odot K_{1,q})$  are cubic graceful.

**KEYWORDS:** , Cubic graceful graph, Cubic graceful labeling, Graceful graph, Star graph .

## I. INTRODUCTION

The vertex set and the edge set of a graph  $G$  denoted by  $V(G)$  and  $E(G)$  respectively. Let  $G(p,q)$  be a graph with  $p = n(G)$  vertices and  $q = m(G)$  edges. Throughout this paper by a graph we mean a finite, connected and undirected with no loops and multiple edges. . A detailed survey of graph labeling can be found in Harray [6].

A function  $f$  is a *graceful labeling* of a graph  $G$  of size  $n$  if  $f$  is an injection from  $V(G)$  to the set  $\{0,1,2,\dots,n\}$  such that , when each edge  $xy$  of  $G$  has assigned the weight  $|f(x)-f(y)|$ , the resulting weights are distinct; in other words, the set of weight is  $\{1,2,\dots,n\}$ . A graph that admits a graceful labeling is said *graceful graphs*.

T. Tharmaraj and P.B.Sarasija [2] introduced *Square Graceful Labeling*.

A graph  $G = (V,E)$  with  $n$  vertices and  $m$  edges is said be **Cubic Graceful Graph (CGG)** if there exists an injective function  $f: V(G) \rightarrow \{0,1,2,\dots,m^3\}$  such that the induced mapping  $f^* : E(G) \rightarrow \{1^3,2^3,\dots,m^3\}$  defined by  $f^*(uv) = |f(u)-f(v)|$  is a bijection . The function  $f$  is called a **Cubic Graceful Labeling (CGL)** of  $G$  [6].

Here we deliberate a new type of cubic labeling of graphs which is closely related to the star graph. .

We use the following definitions in the subsequent sections.

### Definition1.1

A complete bipartite graph  $K_{1,n}$  is called a *star* and it has  $(n+1)$  vertices and  $n$  edges

### Definition1.2

The **Bistar graph  $B_{m,n}$**  is the graph obtained from a copy of a star  $K_{1,m}$  and a copy of star  $K_{1,n}$  by joining the vertices of maximum degree by an edge.

### Definition 1.3

The **Trivial graph  $K_1$  or  $P_1$**  is the graph with one vertex and no edges.

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Vol. 6, Issue 8, August 2017

## II. MAIN RESULTS

### Theorem 2.1

The graph  $pK_{1,q} \cup rK_{1,s}$  is cubic graceful for  $p, q, r, s \geq 1$

**Proof.** Let  $G$  denote the graph  $pK_{1,q} \cup rK_{1,s}$

$$\text{Let } V(G) = \begin{cases} u_m & : 1 \leq m \leq p; \\ u_{mn} & : 1 \leq m \leq p; 1 \leq n \leq q \\ v_t & : 1 \leq t \leq r; \\ v_{tw} & : 1 \leq t \leq r; 1 \leq w \leq s \end{cases}$$

$$\text{And } E(G) = \begin{cases} u_m u_{mn} & : 1 \leq m \leq p; 1 \leq n \leq q \\ v_t v_{tw} & : 1 \leq t \leq r; 1 \leq w \leq s \end{cases}$$

then

$$n(G) = p(1+q) + r(1+s)$$

$$m(G) = pq + rs$$

Define  $f: V(G) \rightarrow \{0, 1, 2, \dots, (pq+rs)^3\}$  by

$$f(u_m) = m-1; \quad 1 \leq m \leq p$$

$$f(u_{1n}) = (pq + rs + 1 - n)^3; \quad 1 \leq n \leq q$$

$$f(u_{mn}) = [q(p+1-m) + rs + 1 - n]^3 + u_m; \quad 2 \leq m \leq p, 1 \leq n \leq q$$

$$f(v_t) = p-1 + t; \quad 1 \leq t \leq r$$

$$f(v_{tw}) = [s(r+1-t) + 1 - w]^3 + v_t; \quad 1 \leq t \leq r, 1 \leq w \leq s$$

The induced edge mapping are

$$f * (u_m u_{mn}) = [q(p+1-m) + rs + 1 - n]^3; \quad 1 \leq m \leq p; 1 \leq n \leq q;$$

$$f * (v_t v_{tw}) = [s(r+1-t) + 1 - w]^3; \quad 1 \leq t \leq r; 1 \leq w \leq s;$$

The vertex labels are arranged in the set  $\{0, 1, 2, \dots, (pq+rs)^3\}$ . Then the edge labels are arranged in the set  $\{1^3, 2^3, \dots, (pq+rs)^3\}$ . Therefore the edge labels are distinct and vertex labels are also distinct. So  $G$  is cubic graceful.

### Theorem 2.2

The graph  $\cup_{r=1}^t K_{1,r}$  is a cubic graceful for all  $t \geq 1$

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**Proof:** Let  $G = \cup_{r=1}^t K_{1,r}$

Let  $V(G) = v_r$  for  $r = 1, 2, \dots, t$   
 $v_{rs}$  for  $r = 1, 2, \dots, t$   $s = 1, 2, \dots, r$

and

$E(G) = v_r v_{rs}$ ; for  $r = 1, 2, \dots, t$   $s = 1, 2, \dots, r$

So

$$n(G) = \frac{t(t+3)}{2} \quad \text{and} \quad m(G) = \frac{t^2+t}{2}$$

Define  $f: V(G) \rightarrow \left\{0, 1, 2, \dots, \left(\frac{t^2+t}{2}\right)^3\right\}$  by

$$f(v_r) = r-1 \quad \text{for } r = 1, 2, \dots, t$$

$$f(v_{11}) = \left(\frac{t^2+t}{2}\right)^3$$

$$f(v_{rs}) = \left[\left(\frac{t^2+t}{2}\right) - \sum_{i=0}^{s-1} (i-s+1)\right]^3 + v_r \quad \text{for } r = 2, 3, \dots, t \quad s = 1, 2, \dots, r$$

The induced edge mapping are

$$f^*(v_r v_{rs}) = \left[\left(\frac{t^2+t}{2}\right) - \sum_{i=0}^{r-1} (i-s+1)\right]^3 \quad \text{for } r = 1, s = 1$$

$$f^*(v_r v_{rs}) = \left[\left(\frac{t^2+t}{2}\right) - \sum_{i=0}^s (i-s+1)\right]^3 \quad \text{for } 2 \leq r \leq t, 1 \leq s \leq r$$

The vertex labels are in the set  $\left\{0, 1, 2, \dots, \left(\frac{t^2+t}{2}\right)^3\right\}$ . Then the edge label arranged in the set  $\left\{1^3, 2^3, 3^3, \dots, \left(\frac{t^2+t}{2}\right)^3\right\}$ . So the vertex labels are distinct and the edge labels are also distinct and cubic. So the graph  $\cup_{r=1}^t K_{1,r}$  is a cubic graceful for all  $t \geq 1$ .

### Theorem 2.3

If  $G_r = K_{1,m}$  for  $1 \leq r \leq n$  be the given graph. Let  $(K_{1,m}; G)$  be the graph obtained from  $r$  copies of  $K_{1,m}$  by joining  $u_{rm}$  with  $u_{(r+1)1}$  for  $1 \leq r \leq n-1$ . Then  $(K_{1,m}; G)$  is cubic graceful graph for all  $m$  and  $n$ .

**Proof:** Let  $(K_{1,m}; G)$  be the given graph,

Let  $V(G) = \{u_r u_{rs}\}; \quad 1 \leq r \leq n, 1 \leq s \leq m$

and

$$E(G) = \left\{ \begin{array}{l} \{u_r u_{rs}\}; \\ \end{array} \right. \quad 1 \leq r \leq n, 1 \leq s \leq m$$

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$$\{u_{rm}u_{(r+1)1}\} \quad 1 \leq r \leq n-1$$

So

$$n(G) = n(m+1) \quad \text{and} \quad m(G) = [n(m+1)-1]$$

Define  $f: V(G) \rightarrow \{0, 1, 2, \dots, (n(m+1)-1)^3\}$  by

$$f(u_1) = 0$$

$$f(u_{1s}) = [n(m+1)-s]^3; \quad 1 \leq s \leq m$$

$$f(u_{rs}) = f(u_{(r-1)3}) - (j)^3; \quad 2 \leq r \leq n; 1 \leq j \leq n-1$$

$$f(u_r) = f(u_1) - [m(n-r+2)]^3; \quad 2 \leq r \leq n$$

$$f(u_{rs}) = f(u_r) + m(n-r+2) - j; \quad 2 \leq r \leq n, 1 \leq s \leq m, 1 \leq j \leq m-1$$

The induced edge mapping are

$$f^*(u_r u_{rs}) = (n(m+1)-j)^3; \quad 1 \leq j \leq mn, 1 \leq r \leq n, 1 \leq s \leq m$$

$$f^*(u_{rm} u_{(r+1)1}) = i^3; \quad 1 \leq r \leq n-1, 1 \leq i \leq n-1$$

The induced edge labels are distinct and they are labeled as  $\{1^3, 2^3, 3^3, \dots, (n(m+1)-1)^3\}$ .

### Theorem 2.4

Let  $K_{1,p} \odot K_{1,q}$  be the given graph G with  $(p+1)(q+1)$  vertices.

**Proof :** Let the vertex set be

$$V(K_{1,p} \odot K_{1,q}) = u, u_r, u_{rs}, w_s; \quad 1 \leq r \leq p, 1 \leq s \leq q$$

and

$$E(K_{1,p} \odot K_{1,q}) = uu_r, u_r u_{rs}, u_r w_s; \quad 1 \leq r \leq p, 1 \leq s \leq q$$

$$\text{Then } n(G) = (p+1)(q+1) \quad \text{and} \quad m(G) = p(q+1) + q$$

Define an injection  $f: V(K_{1,p} \odot K_{1,q}) \rightarrow \{0, 1, 2, \dots, (p(q+1)+q)^3\}$  by

$$f(u) = 0$$

$$f(u_r) = [p(q+1)-r+1]^3; \quad 1 \leq r \leq p$$

$$f(w_s) = [p(q+1)+q-s+1]^3; \quad 1 \leq s \leq q$$

$$f(u_{rs}) = [(p+pq+1)-r+1]^3 - [(r-1)q+s]^3; \quad 1 \leq r \leq p,$$

$$1 \leq s \leq q$$

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Then  $f$  induces a bijection  $f^* : E(K_{1,p} \odot K_{1,q})$

Then,

$$\begin{aligned}
 f^*(u u_r) &= [p(q+1) - r + 1]^3; & 1 \leq r \leq p \\
 f^*(u_r v_{rs}) &= [(r+1)q + s]^3; & 1 \leq r \leq p, 1 \leq s \leq q \\
 f^*(u w_s) &= [(p+1)(q+1) - s]^3; & 1 \leq s \leq q
 \end{aligned}$$

The induced edge labels are distinct and cubic. The edge mappings are  $\{1^3, 2^3, 3^3, \dots, (p(q+1) + q)^3\}$ .

Hence the theorem.

### III. CONCLUSION

The cubic graceful labeling is one of the most important techniques. As all the graphs are not cubic graceful, it is very interesting to investigate graphs or graph families which admit cubic graceful labeling. We have reported the cubic graceful labeling of various graphs.

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